Mind and Motion in Math

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Dear educators:

My project focuses on learning patterning methods to graph equations. Finding rules to tables and multiple representations and how to graph linear and parabolic rules using appropriate scales. We will use kinesthetic activities, visuals, manipulatives (blocks), and group work; take Cornel Notes in our interactive notebooks as well as creating our own representations of what we have learned. We will use problem based learning for deeper understanding. Students will talk about math and decide what a good example of multiple representations and what is not. We will present in front of the class as a group. We will end with movement as we started with movement by doing Math Aerobics.

This works because it is fun and reaches the student in a variety of modalities from movement to visually to being creative. There are many instances when children need to do more than just worksheets and hear us talk. It's about the students taking ownership of their learning and going beyond that with what they have learned. No answers will be given! Thinking and questioning is how we learn. The struggle that we go through to gain information is when the deepest learning happens. Making math fun, interesting and creative is the hook that gets our kids interested and keeps them coming to class for they are wondering what we will be doing!

The following lessons help promote understanding and clarity to the California state standards which is part of the foundation of algebra to build upon not to mention only algebra but for all aspects of mathematics. Any math we learn will help us with deeper understanding, logic and reasoning of a multitude of other aspects of learning especially if it is problem based and the student is part of the learning that goes on.

- Sonya Morris

Lesson Descriptions:

Lesson 1

Walk a graph:

Students will form a human graph while standing on the x-axis outside our classroom on a life size *x-y* axis. They will graph by being directed by the other students who are not on the graph but that are sketching the formation. The group of students they are with they will form various formations by travelling up and down the (y-coordinate). As a class we will discuss what they see forming, identify the *x* and *y* axis, any errors, and if they see a line is it increasing or decreasing.

Interactive Notebook:

As a class we have implemented the use of the interactive notebook. It is an AVID (Advancement via Individual Determination) strategy that I have expanded on. We will take Cornell notes on the right hand page of the notebook and the students express themselves in a creative way on the left side of the notebook. In some cases we will insert information and reflect on it. This is crucial because it involves writing and the students own creative way of expressing what they have learned. This makes their learning more concrete and significant since they have ownership of the creative side and essentially their learning. This helps for greater understanding.

Homework and finish the interactive notebook. Review your notebook.

Lesson 2

Silent Board Game:

The silent board game is played with a table that is missing some of the integers. The object of the game is to be the first group to figure out the pattern. The students discuss and go up and put a number into the table. If it is correct they get points if not they lose points. The whole group has to be in agreement before anyone goes up. The students continue with this until the whole table is filled. The first group to come up with the pattern also acquires points. I then introduce how to find a rule. Then we add that on to the game. It becomes quite competitive and exciting!!!!

Homework and Cornell Notes on graphing correctly with an activity page.
Review your notebook.

Lesson 3

Pattern with graphs:

I will give each group of four a set of blocks to play with first, hexagons to start with. I then let them know that my cousin Mitra is having a wedding and has asked for our help in setting up tables. She wants the table all to be connected sharing one side. Mitra knows that one table will hold 6 people and two will hold ten. She wants to know how many 5, 6, 7, 13, 20 and 100 tables will hold. She wants us to show her some pictures so she can see what it will look like, a graph, table and an algebraic rule.

Mitra also wants us to experiment with a rectangular, triangular, and a square table. We will graph these entirely all on one page to see the similarities and differences.
Finish your graphs and review your notebook.

Lessons 4-5
Patterns that lead to more multiple representations:
Each group of four will be given a pattern of objects on a paper. They will have three to four figures increasing or decreasing in some manner. As a group on their own graph paper the students will have to form the next two objects of the pattern, make an *x-y* table using the figure number and the number of pieces in each figure and draw the graph of the pattern.
This will solidify the concept of how a pattern, table, graph and rule all work together. When they can see this relationship deeper understanding will occur. These ideas also build upon themselves for a meaningful foundation of mathematics.

Develop rubric for Gallery Walk
Homework and review your notebook.

Lesson 6
Gallery walk:
The students will create their own rubric to grade their posters of multiple representations. They will dictate to me what they think should be graded on the poster. I will type up the information they give me and let them do the grading the following day. It might include:

- Is the graph clear and graphed correctly using the graphing rules? ____ 10pts.
- Is the table correct and correlate with the graph, pattern and rule____ 10pts.?
- Is the pattern clearly marked and is the next figure represented? ____ 10pts.
- Does the rule follow all the other components of the graph? ____ 10pts.
- Is the poster neat and cover the whole page? ____ 10pts.
- Over all representation? ____ 10pts.

The students will have their poster up all over the room. Everyone will have a job. The resource manager (gets any materials need for the activity and turns in materials), the facilitator (the team leader who guides the discussion and keep the group on topic), the recorder (writes information down) and a time keeper (keeps the group moving and on task) will be the jobs each person will have to keep the group on task. The students need to discuss and write an explanation of why a poster deserves the certain number of points. One short sentence will work.

It is a beautiful thing to watch. There is an incredible amount of math conversation that occurs. They seem not to notice. Since they came up with the rubric they are hard on each other as far as grading goes.

Homework and Cornell notes on identifying the different kinds of graphs. Review your notebook.

Lesson 7
Work as a group on graphing different equations:

Everyone group will be responsible for graphing one of the given algebraic equations. The students will have to all participate to generate a table, graph, and equation to display in class to help out their fellow classmates.

The students will go up in front of the class to explain their results, as this is happening the other students will sketch the graph that is on their worksheet. Everyone will have to be part of the presentation in some way.

Homework will follow in the way that the students will make a table for the other graphs for homework as long as some more practice. Review your notebook.

Lesson 8
Math aerobics:

Before we test we will review with Algebra Aerobics. It is very similar to "Simon Say" in that the first one that makes a mistake or hesitates has to sit down. The last one standing is the winner.

All the students stand and we go over positive slope or increasing lines, negative slope or decreasing lines, x^2 *etc*…as the students make the motions with their arms and body movements. I let them know the last one standing has an opportunity to earn extra credit points if they are the last five standing and the last one…oh my, possibilities are endless.

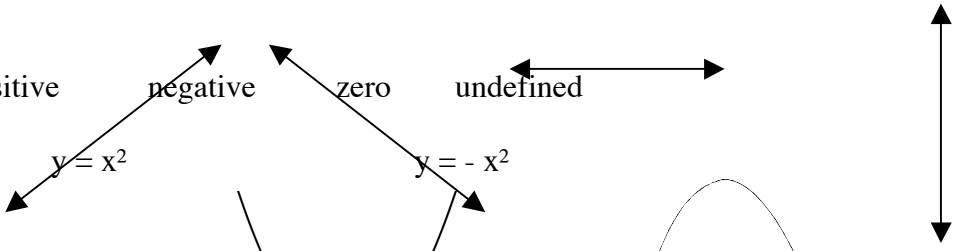
The students love this activity and it is very much worth my time. During the STAR test teachers tell me that my students make the hand movements while sitting so they can remember the graphs.

Comprehensive test on what we have learned.

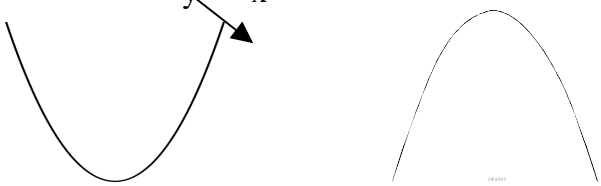
Algebra Aerobics

Use arms and body movements to model the following math terms and the graphs of lines and functions.

SLOPE: positive negative zero undefined

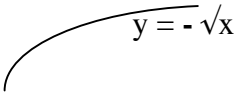


QUADRATIC: $y = x^2$ $y = - x^2$

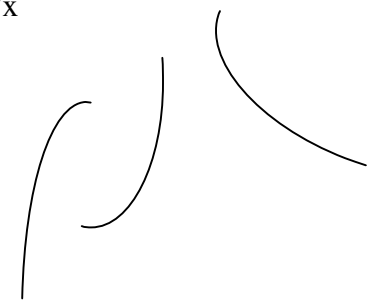


ABSOLUTE VALUE: $y = |x|$ $y = |-x|$

SQUARE ROOT: $y = \sqrt{x}$ $y = - \sqrt{x}$



HYPERBOLA:



Lesson Plans:

Lesson 1
Walk a graph and Interactive Notebook Activity
(Walk a graph is a modified version from CPM)

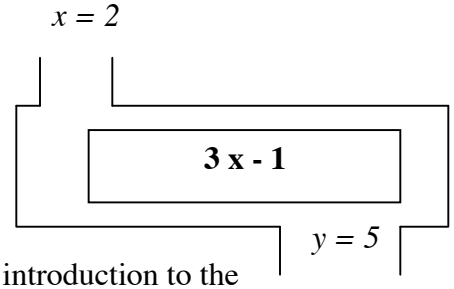
Manipulatives and Resources:

Sidewalk chalk (I prefer this over the rope) or two ropes roughly 25 feet long with x- and y-values marked every 30”
Five sets of colored index cards numbered -6 to 6 (I prefer -10 to 10)
Poster graph paper and sticky dots (if possible dot colors correspond to card colors)
Resource page

The Algebra Walk is from the MCTP professional Development Package, a compilation of classroom- tested ideas from Australian mathematics teachers.

Most students will have some experience with the xy-coordinate system from previous classes. Regardless of a student’s experience our intention is to have the student learn about how the system works by seeing patterns in graphs.

We use the term “input”(x) and “output” (y) to describe the domain and range values. These terms are even more vivid if you tie them to “function machines” where a given value (x) is put into the machine, acted on by the rule, then discharged from the machine as the output of the work. A drawing similar to the one given can represent a function machine



The Algebra walk give students an unconventional introduction to the xy-coordinate system. It is vivid experience that you can refer to through out the project (“Remember when we were outside”). When the outdoor activity is complete, you will have large graphs posted in the classroom to use as the basis for definitions and discussions that the class needs to understand how the xy-coordinate system works. It is important that such teacher – centered activities come after the initial experiences (human graphing) in making the large class graph.

Resist the urge to force formal vocabulary during the first several days of the unit. Let the student come up with the vocabulary themselves. Some will remember it from their previous classes. For example: “left/ right,” “ up/down,” “input/ output” , and “horizontal/ vertical” are actually better to use since they are relatively concrete. Likewise ,comment about “slope” and “intercepts” should stay informal. Using terms such as “ tilt”or “slant”, and “where the line crosses the axis.” The formal vocabulary

will come as you incorporate it into class discussions and when it is presented later in their algebra experience.

By the end of the activity, students should be able to identify the x and y-axis. Reinforce that the line students stand on is the x axis and that students walk parallel to the y-axis.

Have coordinate axis's ready before class. Also have your index cards prepared ahead of time.

Have the poster paper up and ready in the classroom for students to plot (x,y) points after returning from the algebra walk.

Algebra Walk Procedure:

1) Before going out side give each student a data sheet and at least one colored card with one of the integers between -6 and 6 (though I enjoy using -10 and 10 because it gets more kids out on the grid) written on the card. There should be 13 cards for each color, one for each x-value. Each color relates to one of the equations. Each student will also need a pencil for sketching the "human graph" while outdoors.
2) Once outside, situate students so they all are facing the x-axis, looking towards the positive y direction. This orientation is important because it corresponds to the standard orientation we use when graphing.

   Call for students with red cards to find their place along the horizontal axis. The students should stand with both feet on the x-axis facing the positive "y "direction, with their backs to the rest of the class. Start with $y = 2x + 1$. *Have the class give the following directions together to their peers standing on the horizontal axis*. On three have the student call out together, "*Multiply your number by 2 and add one"*. Have students record the resulting number on their resource page. When I say 'Go,' take that number of paces forward or backwards. A 'pace' is the distance between to marks on the vertical axis. Ready? "GO!!!!!"

   Mistakes will be made. Encourage students to help each other out. In most cases, the students will handle corrections themselves. Resist the urge to manage this yourself. Some will cross diagonally and others will let them know.

3) Have the student observers complete the appropriate section of their data sheet. They should roughly sketch and describe each shape they see. Make sure that the students indicate where the line crosses on the x and y – axis with coordinate points (x,y).
4) Repeat this process for each rule on the resource page. You might students that who do the third rule and have them stay in position while others graph the third rule, to introduce the idea of the intersection of lines.

If you want to extend the exercise have a set of students take two steps to their right after they have created a graph of a function. Ask them what features of the

graph change and what features stay the same. This begins an intuitive introduction to translation that will appear occasionally during the year.

5) Back in class: have each group responsible for recording each graph on the poster sheet (poster graph paper) with sticky dots. Have teams of student record their (x,y coordinates) in tables for each rule on the front board, or ask for verbal responses for each separate graph and record the data yourself.

6) Lead a discussion after students complete work that goes along with activity. Talk About (f) and (g) and have students enter this information in their Interactive Note Book.

Doing the problem outside makes this one of the most memorable and enjoyable problems of the year.

The student will enter the data sheet into their Interactive Note book on the left side. They must have three descriptions of each graph after our class discussion. On the right side each student will finish graphing each of the rules providing a table and points of intersection and the written rule once more.

On the bottom of this the students will answer the next two questions:

- Write a paragraph that describes what you did and what you observed in today's class work. Did you notice any patterns? Why was it easy to spot someone who was out of place?
- How can the graph for the rule $y = 2x + 1$ be used to predict the results for na input (x-value) of 7? How can the graph be used to predict the output(y-value) associated with an input of 3 ½
- If you want an output of 7 for the rule $y = -x + 4$, what would you need as an input?
- For each of the rules on the data sheet, where does the graph cross the y- axis? Describe any patterns you notice.

Home work is also included for today as well as another sheet for the interactive note book. "How to set up a graph" They will glue this sheet on the left hand side for tomorrow.

Lesson 2
Silent Board Game:
From CPM
And my own Pattern with graphs:

Manipulatives and resources needed:
Silent board game transparency (optional)

We use the silent board games to introduce the idea of making a table, finding a pattern and writing an equation to solve a problem.

For example: *3x + 1*

½	1	5	2	7	-3	9	0	-6	X
	4	16			-8				

Basic Rules:

1) Students should remain silent during this game. They should raise their hands to be recognized, write an answer on the board, then sit down. This is a game against groups. Each answer that is correct earns their group a point. If it is incorrect the group looses points.
2) Leave correct answers on the board or overhead. Erase the answer if it is incorrect.
3) After each game, students should come up with a rule as a group. Everyone must agree before a hand should be raised. Quiet voices should be used. For an input of "x" we expect a response of" 3x + 1" or "multiply by three and add one" for the game shown above.

Notice that the entries in the top row of the game board are not in acceding order. We want students to find patterns vertically from the input values to the output values in the bottom row. If the input values are in numerical order, it is too easy to find patterns horizontally, and thus miss the purpose of the game, which is to use patterns to discover and write a function rule.

Play two silent board games today, and another one each day to start the day. Here are some examples to use:

3x - 4 x^3 *5 - 2x* *-2x -3* x^2
Silent board games can be used through out the semester to make constructive use of the last few minutes of the class period.

Lesson 3:
Pattern with graphs:

Materials needed:
Regular hexagons, regular triangles, trapezoids, square, and rectangles
Color pencils
Resource page

Students will find this as an easy transition to follow to multiple representations of patterns. They will work within groups and use the manipulatives mentioned, sketching their results as they continue on.

Pattern with Graphs Procedures:

Start the lesson with the following scenario: you have a family member that is getting married and has called on you and your class to solve a problem in setting up tables at her wedding. She wants the table all to be connected sharing one side. The bride knows that one table will hold 6 people and two will hold ten. She wants to know how many 3, 4, 5, 13 and 100 tables will hold. The bride wants us to show her some sketches so she can see how the tables will look, along with a graph, a table and an algebraic rule. She just happens to love math.

Give each group of four a set of blocks to play with first, hexagons to start with. Be sure to give them minute or two (If you don't let them play first they will do it when they are to be working).

Give them the first hand out and let them know to start with the sketch first. They will fill out the table together as a group and continue on to the explanation and the rule. When they finish they will continue on. They will all do this together with a rectangle, triangle, square and trapezoid. When they finish they will graph each of those on the same sheet and answer the questions on the first sheet that was given.

We will discuss the graphs and the questions the following day .

Homework and Cornell Notes on graphing correctly with an activity page.
Review your notebook, finish your graphs.

Lesson 4
Patterns that lead to more multiple representations:

Materials Needed:
A pattern for each group
Resource sheet for multiple representations
Poster paper
Markers
Rulers

Procedure:
We will discuss the results from yesterday's discovery and let them know they are ready for the next step. A picture of a pattern of objects will be give to each group. They will have three to four figures increasing or decreasing in some manner. The students will have to come up with the next two figures of the pattern, make an *x-y* table using the figure number and the number of pieces in each figure, graph and a rule.

This lesson has proved to be a little more challenging for the students because it isn't linear and involves more reasoning. Don't give in but instead lead them to the answer by asking questions. Let the students logically figure it out. Remember when the greatest struggle goes on in learning and then discovery that is when concrete learning takes place and this is what sticks with the students. When they have at least 3 of the four pieces of information then they can start working on their posters. They need to know that it will be graded by their peers.

This will solidify the concept of how a pattern, table, graph and rule all work together. When they can see this relationship deeper understanding will occur. These ideas also build upon themselves for a meaningful foundation of mathematics.

We will work on this project for a day and a half

Gallery walk:
The second half of the following day the students will all participate in creating a rubric to grade our posters of multiple representations. They will dictate to me what they think should be graded on the poster. I will type up the information they give me and let them do the grading the following day. It might include:

Gallery Walk Rubric

- Is the graph clear and graphed correctly using the graphing rules? ____ 10pts.
- Is the table correct and correlate with the graph, pattern and rule? ____ 10pts.
- Is the pattern clearly marked and is the next figure represented? ____ 10pts.
- Does the rule follow all the other components of the graph? ____ 10pts.
- Is the poster neat and cover the whole page? ____ 10pts.
- Over all representation? ____ 10pts.

Homework and review your notebook.

Lesson 5
Gallery walk:

The students will have their poster up all over the room. Everyone will have a job in their groups. The resource manager (gets any materials need for the activity and turns in materials), the facilitator (the team leader who guides the discussion and keep the group on topic), the recorder (writes information down) and a time keeper (keeps the group moving and on task) will be the jobs each person will have to keep the group on task. The students need to discuss and write an explanation of why a poster deserves the certain number of points. One short sentence will work.

It is a beautiful thing to watch. There is an incredible amount of math conversation that occurs. They seem not to notice. Since they came up with the rubric they are hard on each other as far as grading goes.

Take Cornell notes on Identifying different kinds of graphs

Homework and finish upCornell notes on identifying the different kinds of graphs. Review your notebook.

Lesson 6
Work as a group on graphing different equations:

Lesson 7
Math aerobics:

Students will use what they have learned to use their bodies to demonstrate the different graphs they have learned over the past few days. The students will all stand facing the front and together we will go through each graph together. Before we show each one, we will clap and show the answer. For example: arms up in a "u" shape for x^2 and arms in a downward "u" for $- x^2$. Arms held horizontal for "0" slope and straight up and down for undefined. Increasing will be arms held out straight on the sides in a diagonal line going up from left to right. The opposite is shown for decreasing. X^3 can be a little tricky, the right is in an upward half parabola and stand on the right leg while the left leg makes a downward half of a parabola.

Student Impact:
Students will be able to identify lines and graph them knowing what they will look like. The students will be able to discover patterns in numbers as well as objects to come up with a rule. This will lead to understanding how a patterns, tables, graphs and rules are all inter connected. They will know how to distinguish what is a correct graph and what is not as well as converse about math with their peers on why and how the multiple representations of their peers is correctly represented on posters around the room. Student will express what they have learned in words in their interactive notebooks. Students will know how to graph different rules and establish a table for the graphs. In the end they will demonstrate what they have learned with math aerobics and a comprehensive test to assess what they have learned.

Students will learn the necessary Algebra standards in a fun and innovative manner. This will help the students build on what they already know. They will be able to continue on learning the next step in algebra until

Student Assessment:

- I will grade the interactive note books to see if notes were taken correctly.
- They will have a quiz day 5 on what they have learned.
- The students as a group will grade each others multiple representation poster for a grade.
- Culminating test at the end of the unit

5. Standards:

Algebra: 6: Students graph a linear equation and compute the x and y intercepts

7: Students verify that a point lies on a line, given the equation of the line

16: Students understand the concept of a relation and a function.
17: Students determine the domain of the independent variables and the range of dependent variables defined by a graph

21:students graph quadratic functions and know that their roots are the x-intercepts
23: Students apply quadratic equations to physical problems
24: Students use and know simple aspects of a logical argument

Algebra: 1: *Student identify and use the arithmetic properties of subsets of integers and rational and irrational and real numbers.*

2: *students understand and use such operations as taking the opposite, finding the reciprocal, taking a root , and raising to a fractional power*

5: *students solve multistep problems*

9: *students solve a system of two linear equations in two variables algebraically*

and are able to interpret the answer graphically

10*: students add, subtract, multiply and divide monomials and polynomials*.
16: *students understand the concepts of a relation and a function*.

Algebra and Functions: 1.1(7th) Read , write , and compare rational numbers in scientific notation

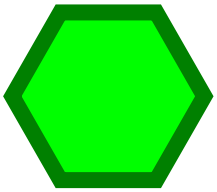
1.2 (7th): Add, subtract, multiply and divide rational numbers

1.5 (7th): Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

Number Sense: 1.0 (7th): students know the properties of, and compute with rational numbers expressed in a variety of forms.

Materials / Budget:

Interactive Notebooks	70 note books @ $5.19	$363.30
Marker sets	9 sets @ $8.00	$ 72
Color pencils	2 sets poster paper for $30.00	$ 30
Side Walk Chalk	1 set @$3.29	$ 3.29
		$ 468.59



Additional:

The Wedding Table Problem

Multiple representations

We have all been called upon to help my cousins with her wedding planning. She wants to know what would be the best shape tables to use. She has options of using a regular hexagon, a regular triangle, a square, and a trapezoid. The catch is that the tables have to be connected side by side and only one person can sit on each side of the table.

What happens to the number of people
en you push two tables together in each
e?

2) How would you find how many people can sit around any number of tables?

Exploration 1:

Use the pattern blocks to explore what happens to the number of people that can sit down when you place the different shapes together end to end. Remember you can only use that one shape repeatedly.

a) What patterns do you notice in your graph?
b) Can you see these patterns in your other representations?
(The table, the picture, the explanation, and the algebraic expression)

Exploration 2:

Investigate what happens when you use different shapes for the tables

a) What are the similarities between the graphs when you use different shaped tables?
b) What are the differences?
c) Can you determine the number of people seated at the 25 tables from your graph? How?

Develop an algebraic expression to give the total number of seats depending on the number of tables (t) placed end-to-end and the number of people (n) able to sit around a table of given sizes.

Graph